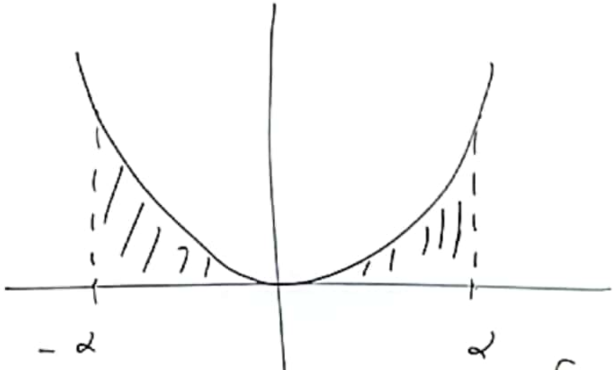
Άσκ.15

α) f άρτια $\Rightarrow \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$



$$\int_{-\alpha}^{\alpha} f(x)dx = \underbrace{\int_{-a}^{0} f(x)dx}_{I_1} + \underbrace{\int_0^a f(x)dx}_{I_2} = 2I_2 \quad \text{ο.ε.δ}$$

Για το I_1 θέτω $x=-u$

x	$-a$	0
u	a	0

$f(-u)=f(u)$
f άρτια

$$I_1 = \int_a^0 f(-u)\cdot(-du) = -\int_{\alpha}^0 f(u)du = \int_0^a f(x)dx = I_2$$

β) f περιττή $\Rightarrow \underbrace{\int_{-\alpha}^{\alpha} f(x)dx}_{I_1} = 0$

$$I_1 = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$$

$$\int_{\alpha}^{-a} f(-u)(-du) = -\int_{-\alpha}^{\alpha} f(u)du$$

$I=-I \to I=0.$

$f(x+T)=f(x) \quad \forall x\in D_f,\ T>0,\ T$ σταθερός

2συν$x+3$ημx ημ$2x$ ημωx $f(x)=$ ημωx, $T=\frac{2\pi}{\omega}$

Αν f περιοδική με περίοδο T.
ν.δ.ο. $\int_{\alpha}^{\alpha+T} f(x)dx = \int_0^T f(x)dx$

(17) $I=\int_0^{\pi/2}\frac{\eta\mu^{\nu}x}{\eta\mu^{\nu}x+\sigma\upsilon\nu^{\nu}x}dx=J=\int_0^{\pi/2}\frac{\sigma\upsilon\nu^{\nu}x}{\eta\mu^{\nu}x+\sigma\upsilon\nu^{\nu}x}=\frac{\pi}{4}$

Θέτω $u=\frac{\pi}{2}-x$ $\quad \eta\mu\left(\frac{\pi}{2}-x\right)=\sigma\upsilon\nu x$

$$I=\int_{\frac{\pi}{2}}^{0}\frac{\eta\mu^{\nu}\left(\frac{\pi}{2}-u\right)}{\eta\mu^{\nu}\left(\frac{\pi}{2}-u\right)+\sigma\upsilon\nu^{\nu}\left(\frac{\pi}{2}-u\right)}\cdot(-du)=\int_0^{\pi/2}\frac{\sigma\upsilon\nu^{\nu}u}{\sigma\upsilon\nu^{\nu}u+\eta\mu^{\nu}u}du \quad \text{ο.ε.δ.}$$

$$I+J=\int_0^{\pi/2}\frac{\eta\mu^{\nu}x+\sigma\upsilon\nu^{\nu}x}{\eta\mu^{\nu}x+\sigma\upsilon\nu^{\nu}x}dx=\int_0^{\pi/2}1\,dx=\frac{\pi}{2}\Rightarrow I+J=\frac{\pi}{2}\Leftrightarrow 2I=\frac{\pi}{2}\Leftrightarrow I=\pi/4$$

(16) $$I=\int_{-\frac{1}{2}}^{\frac{1}{2}}x^3\eta\mu x\cdot\ln\left(\frac{1-x}{1+x}\right)dx=\int_{\frac{1}{2}}^{-\frac{1}{2}}(-u)^3\cdot\eta\mu(-u)\ln\left(\frac{1+u}{1-u}\right)(-du)=$$

$u=-x$

$$=\int_{-\frac{1}{2}}^{\frac{1}{2}}u^3\cdot\eta\mu u\cdot\ln\left(\frac{1+u}{1-u}\right)du=\int_{-\frac{1}{2}}^{\frac{1}{2}}x^3\eta\mu x\cdot\ln\left(\frac{1-x}{1+x}\right)^{-1}dx=-I$$

Άρα $I=0$.

(18) $I=\int_0^{\pi}x\,\eta\mu^5x\,dx=$

(19) $f(x) = x^5 + x^3 + x + 1$. $\quad I = \int_1^4 f^{-1}(x)\,dx$

x	1	4
u	0	1

Θέτω $u = f^{-1}(x) \Rightarrow f(u) = x$

$dx = f'(u)\,du$

$$I = \int_0^1 u\,(5u^4 + 3u^2 + 1)\,du \; \cdots$$

(21) $$\int_0^a x \cdot f''(x)\,dx = \Big[x \cdot f'(x)\Big]_0^a - \int_0^a f'(x)\,dx = a f'(a) - \Big[f(x)\Big]_0^a =$$

$$a \cdot f'(a) - f(a) + f(0) = 0 \Leftrightarrow \frac{\cancel{a} \cdot f'(a)}{\cancel{a}} = \frac{f(a) - f(0)}{a} \Leftrightarrow$$

$f'(a) = f'(\xi) \Rightarrow f'$ όχι $1-1$