ασκ.8

$$I=\int_0^1 \frac{e^{2x}+e^x-3}{e^x+1}\,dx=\int_1^e \frac{u^2+u-3}{u+1}\,\frac{1}{u}\,du=\int_1^e \frac{u^2+u-3}{u^2+u}\,du$$

Θέτω $u=e^x$. $du=e^x dx$

$$=\int_1^e \frac{\cancel{u^2+u}}{\cancel{u^2+u}}\,du-3\int_1^e \frac{1}{u\cdot(u+1)}\,du$$

άσκ.9

α) $$\int_0^{\pi/2}\frac{\eta\mu x}{\sigma\upsilon\nu^2 x+4\sigma\upsilon\nu x+3}\,dx=-\int_1^0 \frac{1}{u^2+4u+3}\,du=\int_0^1 \frac{1}{(u+3)(u+1)}\,du$$

$u=\sigma\upsilon\nu x \Rightarrow du=-\eta\mu x\,dx$

(β) $$\int_0^{\pi/2}\frac{\sigma\upsilon\nu x}{3+\sigma\upsilon\nu^2 x}\,dx=\int_0^{\pi/2}\frac{\sigma\upsilon\nu x}{3+1-\eta\mu^2 x}\,dx=\int_0^1 \frac{1}{4-u^2}\,du$$

(γ) $$\int_0^{\pi/6}\frac{\varepsilon\varphi^2 x+1}{\varepsilon\varphi^2 x-1}\,dx=\int_0^{\sqrt{3}/3}\frac{1}{u^2-1}\,du$$

$u=\varepsilon\varphi x$. $du=\frac{1}{\sigma\upsilon\nu^2 x}\,dx=(1+\varepsilon\varphi^2 x)\,dx$

Άσκ.10

$$\int_{\pi/6}^{\pi/2}\frac{1}{\eta\mu x}\,dx=\int_{\pi/6}^{\pi/2}\frac{\eta\mu x}{\eta\mu^2 x}\,dx=$$

$\frac{1}{\eta\mu x}$ συνεχής στο $[\pi/6,\ \pi/2]$.

$$\int_{\pi/6}^{\pi/2}\frac{\eta\mu x}{1-\sigma\upsilon\nu^2 x}\,dx$$

Θέτω $u=\sigma\upsilon\nu x$

$$\int_0^{\pi/3}\frac{1}{\sigma\upsilon\nu x}\,dx$$

Άσκ. 12 $\quad I_\nu = \int_0^{\pi/2} ημ^\nu x\, dx . \qquad I_\nu = \frac{\nu-1}{\nu} \cdot I_{\nu-2}$

$$I_\nu = \int_0^{\pi/2} ημ^\nu x\, dx = \int_0^{\pi/2} ημ^{\nu-1} x \cdot ημ x\, dx = -\int_0^{\pi/2} ημ^{\nu-1} x \cdot (συν x)' dx =$$

$$= \underbrace{-\left[ημ^{\nu-1} x \cdot συν x\right]_0^{\pi/2}}_{=0} + \int_0^{\pi/2} (\nu-1) \cdot ημ^{\nu-2} x \cdot συν^2 x\, dx = (\nu-1) \cdot \int_0^{\pi/2} \left(ημ^{\nu-2} x - ημ^\nu x\right) dx =$$

$$= (\nu-1)\, I_{\nu-2} - (\nu-1) \cdot I_\nu$$

Άρα $I_\nu = (\nu-1) I_{\nu-2} - (\nu-1) \cdot I_\nu \Leftrightarrow$

$$\nu I_\nu = (\nu-1) I_{\nu-2} \Leftrightarrow \boxed{I_\nu = \frac{\nu-1}{\nu} \cdot I_{\nu-2}}$$

$$I_\nu = \int_0^\pi ημ^\nu x\, dx$$

$$I = \int_0^\pi ημ^5 x\, dx = \int_0^\pi ημ^4 x \cdot ημ x\, dx =$$

$$= \int_0^\pi (1 - συν^2 x)^2 \cdot ημ x\, dx$$

Εφαρμογή:

$$I_8 = \frac{7}{8} I_6$$

$$I_6 = \frac{5}{6} I_4$$

$$I_4 = \frac{3}{4} I_2$$

$$I_2 = \frac{1}{2} I_0$$

$$I_8 \cdot \cancel{I_6} \cdot \cancel{I_4} \cdot \cancel{I_2} = \frac{7}{8} \cdot \frac{5}{\cancel{6}_3} \cdot \frac{\cancel{3}}{4} \cdot \frac{1}{2} \cancel{I_6}\, \cancel{I_4}\, \cancel{I_2}\, I_0$$

$$I_8 = \frac{35}{192} \cdot I_0$$

$$I_0 = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

ασκ.14

$$I = \int_0^{\pi/2} \frac{x \cdot \sigma\upsilon\nu x}{\eta\mu^2 x}\,dx = \int_{\pi/6}^{\pi/2} x \cdot \left(-\frac{1}{\eta\mu x}\right)' dx$$ + παραγοντική.

$$= \int_0^{\pi/2} x\,\frac{\sigma\upsilon\nu x}{1-\sigma\upsilon\nu^2 x}\,dx$$ / Σχολικό 1 – Γενικής / Σελ 234. $\int_0^{\pi} x\cdot f(\eta\mu x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\eta\mu x)\,dx$

Αντικατάσταση $I = \int_a^b f(x)\,dx$ θέτω $u = a+b-x \Rightarrow I = \int_b^a f(a+b-u)(-du)$

$x = a+b-u$

θέτω $u = \frac{\pi}{2} - x$.

$$I = \int_{\pi/2}^{0} \frac{\left(\frac{\pi}{2}-u\right)\cdot\left(\sigma\upsilon\nu\left(\frac{\pi}{2}-u\right)\right)}{\eta\mu^2\left(\frac{\pi}{2}-u\right)}(-du) = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-u\right)\eta\mu u}{\sigma\upsilon\nu^2 u}\,du =$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2}\eta\mu u}{\sigma\upsilon\nu^2 u}\,du - \underbrace{\int_0^{\pi/2} \frac{u\,\eta\mu u}{\sigma\upsilon\nu^2 u}\,du}_{I}.$$ Άρα $2I = \frac{\pi}{2}\cdot\int_0^{\pi/2}\frac{\eta\mu u}{\sigma\upsilon\nu^2 u}\,du$