

S4E

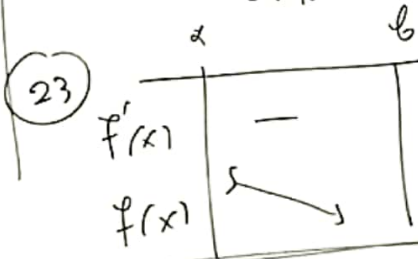
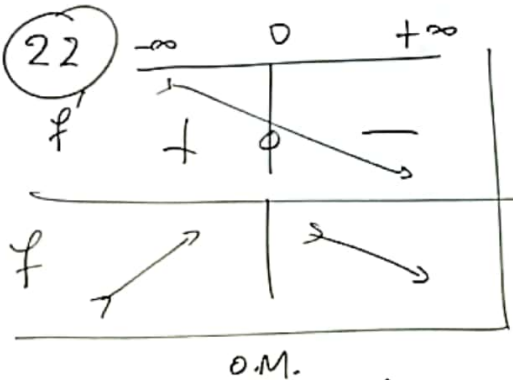
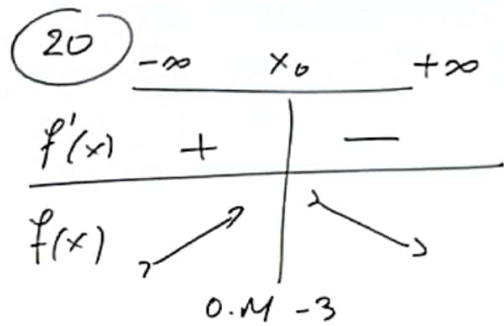
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d)  $I_v = \int_0^1 \frac{x^v}{x^2+1} dx$       $I_v + I_{v-2} = \frac{1}{v-1}$

$$I_v + I_{v-2} = \int_0^1 \frac{x^v}{x^2+1} dx + \int_0^1 \frac{x^{v-2}}{x^2+1} dx =$$

$$= \int_0^1 \frac{x^v + x^{v-2}}{x^2+1} dx = \int_0^1 \frac{x^{v-2} \cdot (x^2+1)}{x^2+1} dx$$

$$= \int_0^1 x^{v-2} dx = \left[ \frac{x^{v-1}}{v-1} \right]_0^1 = \frac{1}{v-1} \quad \text{o.f.s.}$$

b)  $I = \int_0^1 \frac{x^3}{x^2+1} dx = I_3$ ,  $J = \int_0^1 \frac{x^5}{x^2+1} dx$

$$I_3 + I_1 = \frac{1}{3-1} \Leftrightarrow I_3 + I_1 = \frac{1}{2} \cdot (1)$$

$$I_1 = \int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2} \int_0^1 \frac{(x^2+1)'}{x^2+1} dx = \frac{1}{2} \left[ \ln|x^2+1| \right]_0^1 =$$

$$= \frac{1}{2} \ln 2 \cdot (1) \Rightarrow I_3 = \frac{1}{2} - \frac{1}{2} \ln 2 \dots J = I_5$$

$$\Rightarrow \text{es p.w. } I_5 + I_3 = \frac{1}{4}$$

(d)  $I_v < I_{v-2} \Leftrightarrow I_v - I_{v-2} < 0 \Leftrightarrow$

$$\int_0^1 \frac{x^v - x^{v-2}}{x^2+1} dx < 0 \Leftrightarrow \int_0^1 \frac{x^{v-2} \cdot (x^2-1)}{x^2+1} dx < 0$$

for  $x \in (0,1) \rightarrow x^2 < 1 \rightarrow x^2 - 1 < 0$   
 $\hookrightarrow x^{v-2} > 0, x^2+1 > 0 \left\{ \frac{x^{v-2} \cdot (x^2-1)}{x^2+1} < 0 \right.$

$$I_{10} < \frac{1}{18} \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow$$

$$I_{10} + I_8 = \frac{1}{9}$$

$$I_{10} < I_8 \Leftrightarrow I_{10} + I_{10} < I_{10} + I_8 \Leftrightarrow$$

$$2I_{10} < \frac{1}{9} \Leftrightarrow I_{10} < \frac{1}{18}$$

[6u.6] a)  $\int_2^6 x \cdot \sqrt{x-2} dx = \int_0^4 (u+2) \cdot \sqrt{u} du = \int_0^4 u \sqrt{u} du + \int_0^4 2 \sqrt{u} du$

$$= \int_0^4 u^{3/2} du + 2 \int_0^4 u^{1/2} du = \left[ \frac{u^{5/2}}{5/2} \right]_0^4 + 2 \left[ \frac{u^{3/2}}{3/2} \right]_0^4$$

Dim  $u = x-2 \Rightarrow x = u+2$   
 $dx = du$

x	2	6
u	0	4

b)  $J = \int_1^{64} \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx = \int_1^2 \frac{1}{u^3 + u^2} \cdot 6u^5 du = \int_1^2 \frac{6u^3}{u^2(u+1)} du = 6 \int_1^2 \frac{u^3}{u+1} du$

$u = \sqrt[6]{x} \Rightarrow \sqrt{x} = u^3$   
 $\sqrt[3]{x} = u^2$   
 $x = u^6$   
 $dx = 6u^5 du$