S4E

19 → Σ

20 → i → Σ

21 → Σ

22 → i → Σ

23 → Σ

(20)

x	$-\infty$		x_0		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$		↗	ΟΜ-3	↘	

(22)

x	$-\infty$		0		$+\infty$
f'		$+$	0	$-$	
f		↗	ΟΜ	↘	

(23)

x	α		β
$f'(x)$		$-$	
$f(x)$		↘	

Φυλ. | Δευ. 5

α) $I_\nu = \int_0^1 \frac{x^\nu}{x^2+1}\,dx$ $\quad I_\nu + I_{\nu-2} = \frac{1}{\nu-1}$

$$I_\nu + I_{\nu-2} = \int_0^1 \frac{x^\nu}{x^2+1}\,dx + \int_0^1 \frac{x^{\nu-2}}{x^2+1}\,dx =$$

$$= \int_0^1 \frac{x^\nu + x^{\nu-2}}{x^2+1}\,dx = \int_0^1 \frac{x^{\nu-2}\cdot(x^2+1)}{x^2+1}\,dx$$

$$= \int_0^1 x^{\nu-2}\,dx = \left[\frac{x^{\nu-1}}{\nu-1}\right]_0^1 = \frac{1}{\nu-1} \cdot \text{ο.ε.δ}$$

β) $I = \int_0^1 \frac{x^3}{x^2+1}\,dx = I_3$, $\int = \int_0^1 \frac{x^5}{x^2+1}\,dx$

$I_3 + I_1 = \frac{1}{3-1} \Leftarrow 1 \quad I_3 + I_1 = \frac{1}{2} \cdot (1)$

$$I_1 = \int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_0^1 \frac{(x^2+1)'}{x^2+1}\,dx = \frac{1}{2}\left[\ln|x^2+1|\right]_0^1 =$$

$= \frac{1}{2}\ln 2 \cdot (1) \Rightarrow I_3 = \frac{1}{2} - \frac{1}{2}\ln 2 \cdot \cdot \quad \int = I_5$

$\Rightarrow$ έχω $I_5 + I_3 = \frac{1}{4}$

γ) $I_\nu < I_{\nu-2} \Leftrightarrow I_\nu - I_{\nu-2} < 0 \Leftrightarrow$

$$\int_0^1 \frac{x^\nu - x^{\nu-2}}{x^2+1}\,dx < 0 \Leftrightarrow \int_0^1 \frac{x^{\nu-2}\cdot(x^2-1)}{x^2+1}\,dx < 0$$

αν $x \in (0,1) \rightarrow x^2 < 1 \rightarrow x^2 - 1 < 0$ ↳ $x^{\nu-2} > 0$, $x^2+1 > 0$ $\Big\}$ $\frac{x^{\nu-2}\cdot(x^2-1)}{x^2+1} < 0$

$I_{10} < \frac{1}{18}$ ✓

$I_{10} + I_8 = \frac{1}{9}$

$I_{10} < I_8 \Leftrightarrow I_{10} + I_{10} < I_{10} + I_8 \Rightarrow$

$2I_{10} < \frac{1}{9} \Leftrightarrow I_{10} < \frac{1}{18}$

Ασκ. 6 α) $\int_2^6 x \cdot \sqrt{x-2}\, dx = \int_0^4 (u+2)\cdot\sqrt{u}\, du = \int_0^4 u\sqrt{u}\, du + \int_0^4 2\sqrt{u}\, du$

Θέτω $u = x-2 \Rightarrow x = u+2$, $dx = du$

x	2	6
u	0	4

$= \int_0^4 u^{3/2}\, du + 2\int_0^4 u^{1/2}\, du = \left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4 + 2\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

β) $J = \int_1^{64} \frac{1}{\sqrt{x}+\sqrt[3]{x}}\, dx = \int_1^2 \frac{1}{u^3+u^2}\cdot 6u^5\, du = \int_1^2 \frac{6u^5}{u^2(u+1)}\, du = 6\int_1^2 \frac{u^3}{u+1}\, du$

$u = \sqrt[6]{x} \Rightarrow \sqrt{x} = u^3$, $\sqrt[3]{x} = u^2$

$x = u^6$

$dx = 6u^5\, du$