ασκ. 1) (δ)

$$f'(x) = \frac{5}{3x\cdot(\ln x+2)}, \quad x > e^{-2}, \quad f(1/e) = 1$$

$$f'(x) = \frac{5}{3}(\ln x)' \cdot \frac{1}{\ln x + 2} \Leftrightarrow$$

$$f'(x) = \frac{5}{3} \cdot \frac{(\ln x + 2)'}{\ln x + 2} \Leftrightarrow$$

$$\ln x + 2 > 0 \Leftrightarrow \ln x > -2 \Leftrightarrow x > e^{-2}$$

$$f'(x) = \frac{5}{3}\left[\ln(\ln x + 2)\right]'$$

$$f(x) = \frac{5}{3}\ln(\ln x+2) + c.$$

(ε) $f'(x) = εφx - σφ\frac{x}{2}$, $x \in (0, \pi/2)$, $f\left(\frac{\pi}{3}\right) = 3\ln$

$$εφx = \frac{ημx}{συνx} = -\frac{(συνx)'}{συνx} = -(\ln|συνx|)'$$

$$σφx = \frac{συνx}{ημx} = \frac{(ημx)'}{ημx} = (\ln|ημx|)'$$

$$σφ\frac{x}{2} = \left(2\cdot\ln\left|ημ\tfrac{x}{2}\right|\right)'$$

(στ) $f'(x) = \frac{1}{e^x+1}$, $x \in \mathbb{R}$, $f(0) = 1$

$$f'(x) = \frac{e^x+1-e^x}{e^x+1} = \frac{e^x+1}{e^x+1} - \frac{e^x}{e^x+1} = 1 - \frac{(e^x+1)'}{e^x+1}$$

$$f'(x) = \frac{1}{e^x+1} = \frac{e^{-x}}{1+e^{-x}} = -\frac{(e^{-x})'}{1+e^{-x}} = -\frac{(1+e^{-x})'}{1+e^{-x}}$$

ζ) $f'(x) = \frac{1}{x^2+x} = \frac{1}{x\cdot(x+1)} = \frac{α}{x} + \frac{β}{x+1}$

ασκ. 2

α) $f(x)=\dfrac{1}{\sqrt{x+1}-\sqrt{x}}$

$x\geq 0$, $x+1\geq 0$

$\sqrt{x+1}-\sqrt{x}\neq 0 \Leftrightarrow$

$\sqrt{x+1}\neq\sqrt{x} \Leftrightarrow 1\neq 0$

$[0,+\infty)$

$f(x)=\dfrac{\sqrt{x+1}+\sqrt{x}}{x+1-x} \Leftrightarrow$

$f(x)=\sqrt{x+1}+\sqrt{x}$

$=(x+1)^{1/2}+x^{1/2}$

$=\left[\frac{2}{3}(x+1)^{3/2}+\frac{2}{3}x^{3/2}\right]'$

Παράγουσες $\frac{2}{3}\sqrt{(x+1)^3}+\frac{2}{3}\sqrt{x^3}+c$

$x\in[0,+\infty)$

$\sqrt{x}=x^{1/2}\longrightarrow\frac{2}{3}x^{\frac{3}{2}}$

$\sqrt{f(x)}\cdot f'(x)=\frac{2}{3}\left(f(x)\right)^{3/2}$

$f(x)=\dfrac{x}{\sqrt{x^2+1}+\sqrt{x^2+2}}$, $x\in\mathbb{R}$

$=\dfrac{x\cdot\left(\sqrt{x^2+1}-\sqrt{x^2+2}\right)}{\cancel{x^2}+1-\cancel{x^2}-2}=$

$=x\cdot\sqrt{x^2+2}-x\cdot\sqrt{x^2+1}$

$=\frac{1}{2}(x^2+2)'\sqrt{x^2+2}-\frac{1}{2}(x^2+1)'\cdot\sqrt{x^2+1}=$

$=\frac{1}{2}\cdot\frac{2}{3}\cdot\left[(x^2+2)^{3/2}\right]'-\frac{1}{\cancel{2}}\cdot\frac{2}{3}\left[(x^2+1)^{3/2}\right]'$

σ) $\int \frac{e^{3x}+1}{e^x+1}\,dx = \int \frac{e^{3x}+1}{e^x\cdot(e^x+1)}\cdot e^x\,dx = \int \frac{u^3+1}{u^2+u}\,du =$

$x \in \mathbb{R} \quad e^x = u \Rightarrow du = e^x\,dx$

$= \int \frac{\cancel{(u+1)}\cdot(u^2-u+1)}{u\cdot\cancel{(u+1)}}\,du =$

$e^{3x} + 0e^{2x} + 0\cdot e^x + 1 \;\big|\; e^x+1$

$= \int \left(\frac{u^2}{u} - \frac{u}{u} + \frac{1}{u}\right)du = \int \left(u - 1 + \frac{1}{u}\right)du$

$(e^x)^3 + 1$

$= \frac{u^2}{2} - u + \ln|u| = \frac{e^{2x}}{2} - e^x + \ln e^x + C$

(ε) $\frac{4^x+1}{2^x+1} = \frac{(2^x)^2+1}{2^x+1} = \frac{(2^x)^2+2\cdot 2^x+1}{2^x+1} - \frac{2\cdot 2^x}{2^x+1} = \frac{(2^x+1)^{\cancel{2}}}{\cancel{2^x+1}} - 2\,\frac{2^x}{2^x+1}$

$2^{2x} = (2^x)^2 + 1$

$\frac{2^x}{2^x+1} = \frac{\left(\frac{1}{\ln 2}\cdot 2^x\right)'}{2^x+1}$