

αβ.5) $3x^4 + x + 2 \ln x = 4$

Προφανώς λύση $x = 1$.

Έστω $A(x) = 3x^4 + x + 2 \ln x$

$A'(x) = 12x^3 + 1 + \frac{2}{x} > 0$

$\forall x > 0 \Rightarrow A(x) \uparrow$ στο $(0, +\infty)$

Από 1-1 \Rightarrow η λύση μοναδική.

$(\ln x)' = \frac{1}{x}$ $(\ln|x|)' = \frac{1}{x}$

$\forall x > 0$

$\Delta < 0 \Rightarrow$ $f'(x)$ ομογ. τον a

αβ.6) $f(x) = \frac{e^{ax}}{x^2 + a^2}$, $D_f = \mathbb{R}$.

$f'(x) = \frac{e^{ax} \cdot a \cdot (x^2 + a^2) - e^{ax} \cdot (2x)}{(x^2 + a^2)^2} =$

$= \frac{e^{ax} (ax^2 + a^3 - 2x)}{(x^2 + a^2)^2} = \frac{e^{ax} (ax^2 - 2x + a^3)}{(x^2 + a^2)^2} +$

$\Delta = 4 - 4 \cdot a^4 = 4 \cdot (1 - a^4)$

$\Delta > 0 \Rightarrow 1 - a^4 > 0 \Rightarrow a^4 < 1 \Rightarrow |a| < 1 \Rightarrow -1 < a < 1$, αλλ

$\Delta < 0 \Rightarrow a \in (-\infty, -1) \cup (1, +\infty)$, $\Delta = 0 \Rightarrow a = \pm 1$

Αν $a > 0$ $\frac{f'(x)}{f(x)} +$

$\hookrightarrow a \geq 1$

Αν $a < 0$ $\frac{f'(x)}{f(x)} -$

$a \leq -1$

abu.6 (B)

i) $x \rightarrow \infty \rightarrow e^x > x^2 + 1$

$f(x) = \frac{e^{ax}}{x^2 + a^2}$, $\begin{cases} a \geq 1 \Rightarrow f \uparrow \\ a \leq -1 \Rightarrow f \downarrow \end{cases}$

für $a=1 \Rightarrow f(x) = \frac{e^x}{x^2 + 1} \uparrow$ für \mathbb{R}

$f(0) = \frac{e^0}{1} = 1$

$\forall x > 0 \Rightarrow f(x) > f(0) = 1$

$\frac{e^x}{x^2 + 1} > 1 \Rightarrow \dots$

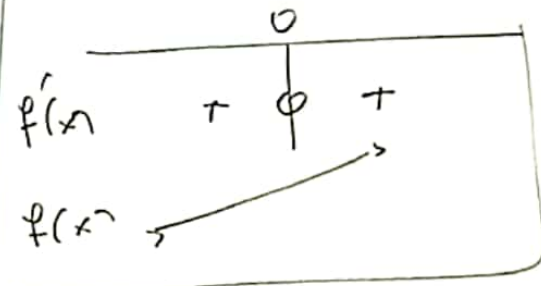
abu.8

$f(x) = e^x - e^{-x} + 2 \ln x, x \in \mathbb{R}$

f. GWS für \mathbb{R} .

$f'(x) = e^x + e^{-x} + 2 \ln x = \left(e^x + \frac{1}{e^x} \right) + 2 \ln x$

Given $e^x + \frac{1}{e^x} \geq 2$ for $x > 0$ with "=" holds for $x=1$
 $2 \geq 2 \ln x$ with "=" holds for $x=2$



$\lim_{x \rightarrow +\infty} f(x) =$

$\lim_{x \rightarrow +\infty} (e^x) = +\infty, \lim_{x \rightarrow +\infty} e^{-x} = 0$

$-2 \leq 2 \ln x \leq 2 \Leftrightarrow e^x - e^{-x} - 2 \leq f(x) \leq 2 + e^x - e^{-x}$
 $\lim_{x \rightarrow +\infty} (e^x - e^{-x} - 2) = +\infty \Rightarrow \lim_{x \rightarrow +\infty} f(x) = +\infty$