άσκ 8/ $f(x) = ημx$, $g(x) = συνx$, $x_0 = π/4$

$f'(x) = συνx$. $f'(\frac{π}{4}) = \frac{\sqrt{2}}{2}$. $ε_1: \ y - f(π/4) = f'(π/4)\cdot(x - π/4) \Leftrightarrow$

$$\Leftrightarrow y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\cdot\left(x - \frac{π}{4}\right)$$

$g'(x) = -ημx$. $g'(π/4) = -\frac{\sqrt{2}}{2}$. $ε_2: \ y - g(π/4) = g'(π/4)\cdot(x - π/4) \Leftrightarrow$

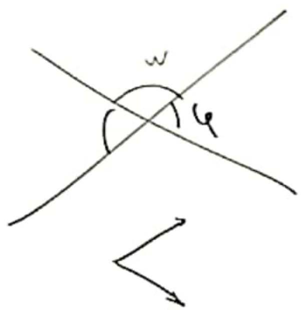
$$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{π}{4}\right)$$

(β) $λ_{ε_1} = \frac{\sqrt{2}}{2}$. ένα διάνυσμα $\vec{δ}_1 \parallel ε_1$ είναι $\vec{δ}_1 = \left(1, \frac{\sqrt{2}}{2}\right)$

$λ_{ε_2} = -\frac{\sqrt{2}}{2}$ " " $\vec{δ}_2 \parallel ε_2$ " $\vec{δ}_2 = \left(1, -\frac{\sqrt{2}}{2}\right)$

$$\vec{δ}_1\cdot\vec{δ}_2 = |\vec{δ}_1|\cdot|\vec{δ}_2|\cdot συνφ \Leftrightarrow συνφ = \frac{\vec{δ}_1\cdot\vec{δ}_2}{|\vec{δ}_1|\cdot|\vec{δ}_2|} = \frac{1\cdot 1 + \frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)}{\sqrt{1^2 + \frac{\sqrt{2}^2}{2^2}}\cdot\sqrt{1^2 + \left(-\frac{\sqrt{2}}{2}\right)^2}} =$$

$$= \frac{1 - \frac{2}{4}}{\cancel{\sqrt{1+\frac{2}{4}}}^{\,2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}.$$



$συνφ = \frac{1}{3}$

$συνω = -\frac{1}{3}$

Αν $\vec{δ} = (x_1, y_1)$

1) $λ_{\vec{δ}} = \frac{y_1}{x_1}$

2) $\vec{δ} \parallel ε \Leftrightarrow λ_{\vec{δ}} = λ_ε$

ασκ 9)

i) f' συνεχής στο 0

ii) $f(0)=1$

iii) $\lim_{x\to 0} \frac{2f'(x)-6f(x)+ημx}{x} = 3$

(α) Θέτω $A(x) = \frac{2f'(x)-6f(x)+ημx}{x}$

$$2f'(x) = x\cdot A(x)+6f(x)-ημx$$

$$f'(x) = \frac{x\cdot A(x)+6f(x)-ημx}{2}$$

$$\lim_{x\to 0} \frac{x\cdot A(x)+6f(x)-ημx}{2} = \frac{0\cdot 3+6\cdot 1-0}{2} = 3$$

Άρα $\lim_{x\to 0} f'(x) = 3$, αφού f' συνεχής στο 0 $\Rightarrow f'(0)=3$.

(β) Θα αναζητήσω το $\lim_{x\to 0} \frac{f'(x)-f'(0)}{x} = \lim_{x\to 0} \frac{f'(x)-3}{x} = \lim_{x\to 0} \frac{\frac{xA(x)+6f(x)-ημx}{2}-3}{x} =$

$$\lim_{x\to 0} \frac{x\cdot A(x)+6f(x)-ημx-6}{2x} = \left[\lim_{x\to 0} \frac{\cancel{x}\cdot A(x)}{2\cancel{x}} + \lim_{x\to 0} \frac{6f(x)-6}{2x} - \lim_{x\to 0} \frac{ημx}{2x}\right] =$$

$$= \frac{3}{2} + 3\cdot \lim_{x\to 0} \frac{f(x)-1}{x} - \frac{1}{2} = \frac{3}{2} + 3\cdot f'(0) - \frac{1}{2} = \frac{3}{2} + 3\cdot 3 - \frac{1}{2} = 10$$

Άρα $\lim_{x\to 0} \frac{f'(x)-f'(0)}{x} = 10 \Rightarrow f''(0)=10$.

ασκ. 10

$\varepsilon: y = x + \beta$. , $f(x) = \ln x$

Αν $A(x_0, \ln x_0)$ το σημείο επαφής

$\Rightarrow f'(x_0) = 1$
όμως $f'(x) = \frac{1}{x}$ $\Big\} \rightarrow \frac{1}{x_0} = 1 \Leftrightarrow x_0 = 1$

Άρα $A(1, \ln 1) = (1, 0)$

$\varepsilon: y - 0 = 1\cdot(x-1) \Leftrightarrow y = x - 1 \Rightarrow \beta = -1$

♡ $f'(x)\cdot g'(x) = c \quad \forall x \in \mathbb{R}$

$h(x) = f(x)\cdot g(x) \rightarrow$

$h'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$h''(x) = f''(x)\cdot g(x) + f'(x)\cdot g'(x) + f'(x)\cdot g'(x) + f(x)\cdot g''(x)$

$h''(x) = f''(x)\cdot g(x) + 2c + f(x)\cdot g''(x)$

$h'''(x) = f'''(x)\cdot g(x) + f''(x)\cdot g'(x) + f'(x)\cdot g''(x) + f(x)\cdot g'''(x) \Rightarrow$

$$\frac{h'''(x)}{h(x)} = \frac{f'''(x)\cdot g(x)}{f(x)\cdot g(x)} + \frac{f''(x)\cdot g'(x) + f'(x)\cdot g''(x)}{f(x)\cdot g(x)} + \frac{f(x)\cdot g'''(x)}{f(x)\cdot g(x)} \Leftrightarrow$$

$$\frac{h'''(x)}{h(x)} = \frac{f'''(x)}{f(x)} + \frac{g'''(x)}{g(x)}$$

Από τη σχέση: $f'(x)\cdot g'(x) = c \Rightarrow$

$f''(x)\cdot g'(x) + f'(x)\cdot g''(x) = 0$