$(x^{\nu})' = \nu \cdot x^{\nu-1}$. Έστω $f(x) = x^{\nu}$, $x \in \mathbb{R}$. $\nu \geq 1$, $x' = 1$, $c' = 0$

Έστω τυχαίο $x_0 \in \mathbb{R}$. Ζητάω το $\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{x^{\nu} - x_0^{\nu}}{x - x_0} =$

$$= \lim_{x \to x_0} \frac{\cancel{(x - x_0)} \cdot (x^{\nu-1} + x^{\nu-2} \cdot x_0 + x^{\nu-3} \cdot x_0^2 + \dots + x_0^{\nu-1})}{\cancel{x - x_0}} = \underbrace{x_0^{\nu-1} + x_0^{\nu-1} + x_0^{\nu-1} + \dots + x_0^{\nu-1}}_{\nu \cdot x_0^{\nu-1}}$$

$f'(x_0) = \nu \cdot x_0^{\nu-1}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$, $x \in (0, +\infty)$

Έστω $f(x) = \sqrt{x}$. Έστω $x_0 \in (0, +\infty)$.

$$\lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{x - x_0} = \lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{\sqrt{x}^2 - \sqrt{x_0}^2} = \lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{(\sqrt{x} - \sqrt{x_0})(\sqrt{x} + \sqrt{x_0})} = \frac{1}{2\sqrt{x_0}}$$

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$ $f(x) = \frac{1}{x}$. $x_0 \in \mathbb{R}^*$

$$\lim_{x \to x_0} \frac{\frac{1}{x} - \frac{1}{x_0}}{x - x_0} = \lim_{x \to x_0} \frac{\frac{x_0 - x}{x \cdot x_0}}{\frac{x - x_0}{1}} = \lim_{x \to x_0} \frac{-\cancel{(x - x_0)} \cdot 1}{x \cdot x_0 \cdot \cancel{(x - x_0)}} = -\frac{1}{x_0^2}.$$

$(\eta\mu x)' = \sigma\upsilon\nu x$

$(\sigma\upsilon\nu x)' = -\eta\mu x$

$(e^x)' = e^x$

$(\varepsilon\varphi x)' = -\frac{1}{\sigma\upsilon\nu^2 x}$

$(\ln x)' = \frac{1}{x}$, $x > 0$

$(\sigma\varphi x)' = \frac{1}{\eta\mu^2 x}$

$$f(x)=\begin{cases} x^2+x+a^2, & x<0 \\ x^3+ax+1, & x\geq 0 \end{cases}$$

Πρέπει f συνεχής στο $x_0=0 \rightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0) \Leftrightarrow$

$a^2=1 \Leftrightarrow a=\pm 1$

Για να είναι παρ/μη στο $x_0=0 \Leftrightarrow$

$$\lim_{x\to 0^-}\frac{(x^2+x+a^2)-1}{x} = \lim_{x\to 0^+}\frac{x^3+ax+1-1}{x} \Leftrightarrow$$

$$\lim_{x\to 0^-}\frac{x^2+x+\cancel{1}-\cancel{1}}{x} = \lim_{x\to 0^+}\frac{x^3+ax}{x} \Leftrightarrow$$

$$\lim_{x\to 0^-}\frac{2x+1}{1} = \lim_{x\to 0^+}\frac{3x^2+a}{1} \Leftrightarrow \boxed{1=a}$$