

ΘΜΕΤ. αλμ. ↓ $f(0) = \eta, f(1) = \epsilon. g(x) = 2 + |f(x) - 3|$

Αποδ. $|f(x) - 3| \geq 0 \iff 2 + |f(x) - 3| \geq 2.$

Α.ν.δ.ο. $\exists x_1 \text{ ώστε } g(x_1) = 2 \iff f(x_1) - 3 = 0 \iff f(x_1) = 3.$

Αγών f συνεχ. στο $[0, 1]$ } \rightarrow Αποδ. $\forall \epsilon \tau.$ υπάρχει

$f(0) \neq f(1)$

$x_1 \in (0, 1)$ ώστε $f(x_1) = 3$ ο.ε.δ.

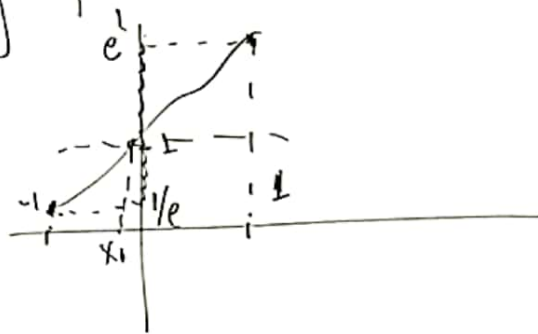
$e < 3 < \eta$

αλμ. 2 $A(1, e^2), B(-1, 1/e)$

$x) g(x) = f^2(x) - 2f(x) + 3 = f(x)^2 - 2f(x) + 1 + 2 = (f(x) - 1)^2 + 2 \geq 2$

$1 \in (1/e, e^2)$

f συνεχ. στο $[-1, 1]$



$f(x_1) = 1$

$$A(1, e^2), B(-1, 1/e)$$

$$b) h(x) = -2f^2(x) + 4f(x) = -2 \cdot (f^2(x) - 2f(x)) = -2 \cdot (f^2(x) - 2f(x) + 1 - 1) =$$

$$= -2 \cdot (f(x) - 1)^2 + 2$$

$$(f(x) - 1)^2 \geq 0 \Leftrightarrow -2(f(x) - 1)^2 \leq 0 \Leftrightarrow -2(f(x) - 1)^2 + 2 \leq 2$$

$$\text{0.0.0. } \exists x_1 \text{ woz } h(x_1) = 2 \Leftrightarrow f(x_1) = 1 \dots$$

∪ ∩

2nd method. Gehe $A(x) = -2x^2 + 4x$, dann $h(x) = A(f(x))$

$$\text{H } A(x) \text{ nimmt in } \mathbb{R} \text{ sein Maximum bei } x_1 = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$$

$$A(x) \leq A(1) \quad \forall x \in \mathbb{R} \Rightarrow A(f(x)) \leq A(1) \quad \forall x \in \mathbb{R}$$

$$\text{Daher ist } \exists x_1 : f(x_1) = 1 \text{ (?)} \Rightarrow A(f(x)) \leq A(f(x_1)) \Leftrightarrow h(x) \leq h(x_1)$$

$$(8) \varphi(x) = 2 - 3 \ln\left(\frac{1}{4} f(x)\right)$$

$$-1 \leq \ln\left(\frac{1}{4} f(x)\right) \leq 1 \Leftrightarrow -3 \leq -3 \ln\left(\frac{1}{4} f(x)\right) \leq 3 \Leftrightarrow -1 \leq 2 - 3 \ln\left(\frac{1}{4} f(x)\right) \leq 5$$

$$\text{0.0.0. } \exists x_1 : \varphi(x_1) = 5 \Leftrightarrow \ln\left(\frac{1}{4} f(x_1)\right) = -1 \Leftrightarrow \frac{1}{4} f(x_1) = \frac{3}{2} \Leftrightarrow f(x_1) = 6 \in \left(\frac{1}{e}, e^2\right)$$

$$\exists x_2 : \varphi(x_2) = -1 \Leftrightarrow \ln\left(\frac{1}{4} f(x_2)\right) = 1 \Leftrightarrow \frac{1}{4} f(x_2) = \frac{3}{2} \Leftrightarrow f(x_2) = 6 \in \left(\frac{1}{e}, e^2\right)$$

$$\frac{1}{4} f(x_2) = \frac{1}{2} \Leftrightarrow f(x_2) = 2 \in \left(\frac{1}{e}, e^2\right)$$