ασκ. 1

$$f(x)=\begin{cases} x^2-8x+16, & 0<x<5 \\ (\alpha^2+\beta^2)\cdot\ln(x-5+e)+2\cdot(\alpha+1)\cdot e^{5-x}, & x\geq 5 \end{cases}$$

α) $\lim\limits_{x\to 5^-} f(x) = \lim\limits_{x\to 5^-}(x^2-8x+16) = 25-40+16 = 1$

$\lim\limits_{x\to 5^+} f(x) = (\alpha^2+\beta^2)\cdot\ln e + 2\cdot(\alpha+1)\cdot e^{0} = \alpha^2+\beta^2+2\alpha+2$

β) Η f είναι συνεχής στο $x_0=5 \iff$

$\lim\limits_{x\to 5^-} f(x) = \lim\limits_{x\to 5^+} f(x) = f(5) \iff$

$\alpha^2+\beta^2+2\alpha+2 = 1 \iff \alpha^2+2\alpha+1+\beta^2 = 0 \iff (\alpha+1)^2+\beta^2 = 0 \iff$

$\alpha+1=0$ και $\boxed{\beta=0}$

$\boxed{\alpha=-1}$

$x \geq 5 \qquad f(x) = \ln(x-5+e)$

$$\lim_{x\to+\infty}(x-5+e) = +\infty \Rightarrow \lim_{x\to+\infty}\left[\ln(x-5+e)\right] = \lim_{u\to+\infty} \ln u = +\infty$$

$u = x-5+e$

$\underline{\alpha\sigma\kappa 2}$

$$f(x) = \begin{cases} \dfrac{x^2+bx+a}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$$

Πρέπει και αρκεί $\boxed{\lim_{x\to1} \underbrace{\dfrac{x^2+bx+a}{x-1}}_{A(x)} = 3} \quad (1)$

Αφού $\lim_{x\to1}(x-1) = 0$, $\lim_{x\to1} A(x) = 3 \in \mathbb{R}$

από γνωστό λήμμα $\Rightarrow \lim_{x\to1}(x^2+bx+a) = 0 \Leftrightarrow a+b+1=0$

$\boxed{a = -b-1 \quad (2)}$

$$(1) \Rightarrow \lim_{x\to1}\frac{x^2+bx-b-1}{x-1} = 3 \Leftrightarrow \lim_{x\to1}\frac{(x-1)\cdot(x+1)+b(x-1)}{x-1} = 3 \Leftrightarrow$$

$$\Leftrightarrow \lim_{x\to1}\frac{\cancel{(x-1)}\cdot(x+1+b)}{\cancel{x-1}} = 3 \Leftrightarrow 2+b = 3 \Leftrightarrow \boxed{b=1} \Rightarrow \boxed{a=-2}$$

αση 3) $f^2(x)+g^2(x)= x^4-2x^2+1 \quad (1)$

για $x=1 \Rightarrow f^2(1)+g^2(1)=0 \longrightarrow f(1)=g(1)=0.$

$\lim_{x\to 1}(x^4-2x^2+1)=0 \overset{(1)}{\Rightarrow} \lim_{x\to 1}\left(f^2(x)+g^2(x)\right)=0$

$0 \leq f^2(x) \leq f^2(x)+g^2(x) - \lim_{x\to 1} f^2(x)=0 \longrightarrow \lim_{x\to 1}|f(x)|=\lim_{x\to 1}\sqrt{f^2(x)}=0$

$-|f(x)| \leq f(x) \leq |f(x)|$

$\to 0 \leftarrow$