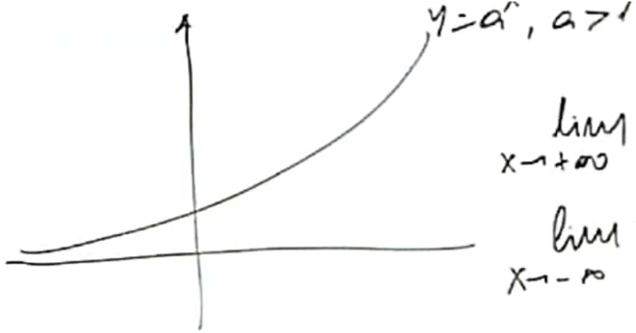
ασκ14 / Εκθετική & λογαριθμική.

(γ) $\lim\limits_{x\to+\infty} \left(\dfrac{3^x + a^{x-1}}{2^x + 4^{x+1}}\right)$ $A(x)$

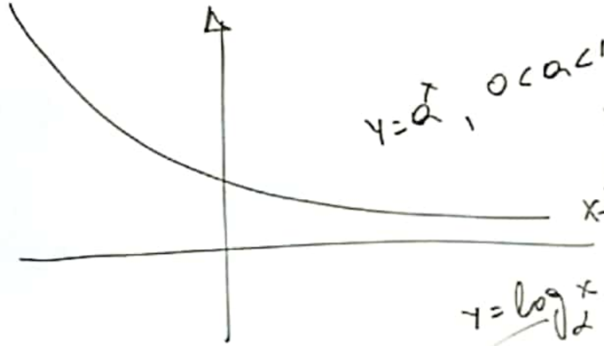


$\lim\limits_{x\to+\infty} a^x = +\infty$

$\lim\limits_{x\to-\infty} a^x = 0$

(1ον) Αν $a>3$

$$A(x) = \frac{a^x \cdot \left(\left(\frac{3}{a}\right)^x + a^{-1}\right)}{4^x \cdot \left(\left(\frac{2}{4}\right)^x + 4\right)}$$



$\lim\limits_{x\to+\infty} a^x = 0$

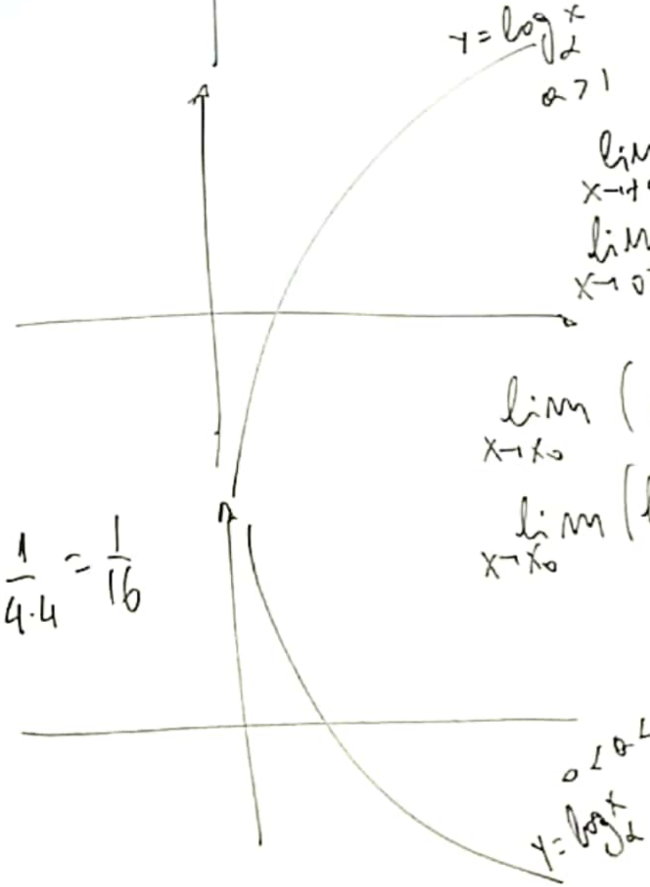
$\lim\limits_{x\to-\infty} a^x = +\infty$

$\lim\limits_{x\to+\infty} \left(\left(\frac{3}{a}\right)^x + \frac{1}{a}\right) = \frac{1}{a}$

$\lim\limits_{x\to+\infty} \left(\left(\frac{2}{4}\right)^x + 4\right)$.

(1α) Αν $a>4 \Rightarrow A(x) = \left(\frac{a}{4}\right)^x \cdot \dfrac{\left(\frac{3}{a}\right)^x + a^{-1}}{\left(\frac{2}{4}\right)^x + 4}$

$\lim\limits_{x\to+\infty} \left(\frac{a}{4}\right)^x = +\infty$, $\lim\limits_{x\to+\infty} A(x) = +\infty$



$\lim\limits_{x\to+\infty} f(x) = +\infty$

$\lim\limits_{x\to0^+} f(x) = -\infty$

$\lim\limits_{x\to x_0} \left(a^x\right) = a^{x_0}$

$\lim\limits_{x\to x_0} \left(\log_a x\right) = \log_a x_0$

(1β) Αν $a=4 \Rightarrow A(x) = \dfrac{\left(\frac{3}{a}\right)^x + \frac{1}{a}}{\left(\frac{2}{4}\right)^x + 4}$. $\lim\limits_{x\to+\infty} A(x) = \frac{1}{4\cdot 4} = \frac{1}{16}$

(1γ) Αν $3<a<4$ $\Rightarrow \lim\limits_{x\to+\infty} \left(\frac{a}{4}\right)^x = 0 \Rightarrow \lim\limits_{x\to+\infty} A(x) = 0$

ασκ14 / Εκθετική & λογαριθμική.

(γ) $\lim\limits_{x\to+\infty} \left(\dfrac{3^x + a^{x-1}}{2^x + 4^{x+1}} \right)$ $A(x)$

1ον Αν $a > 3$

$$A(x) = \frac{a^x \cdot \left(\left(\frac{3}{a}\right)^x + a^{-1} \right)}{4^x \cdot \left(\left(\frac{2}{4}\right)^x + 4 \right)}$$

$\lim\limits_{x\to+\infty} \left(\left(\frac{3}{a}\right)^x + \frac{1}{a} \right) = \frac{1}{a}$

$\lim\limits_{x\to+\infty} \left(\left(\frac{2}{4}\right)^x + 4 \right)$.

1α Αν $a > 4 \Rightarrow A(x) = \left(\frac{a}{4}\right)^x \cdot \dfrac{\left(\frac{3}{a}\right)^x + a^{-1}}{\left(\frac{2}{4}\right)^x + 4}$

$\lim\limits_{x\to+\infty} \left(\frac{a}{4}\right)^x = +\infty$, $\lim\limits_{x\to+\infty} A(x) = +\infty$

1β Αν $a = 4 \Rightarrow A(x) = \dfrac{\left(\frac{3}{a}\right)^x + \frac{1}{a}}{\left(\frac{2}{4}\right)^x + 4}$. $\lim\limits_{x\to+\infty} A(x) = \frac{1}{4\cdot 4} = \frac{1}{16}$

1γ Αν $3 < a < 4$ $\Rightarrow \lim\limits_{x\to+\infty} \left(\frac{a}{4}\right)^x = 0 \Rightarrow \lim\limits_{x\to+\infty} A(x) = 0$

(2) Αν $a = 3$. $A(x) = \dfrac{3^x \left(1 + \frac{1}{3}\right)}{4^x \left(\left(\frac{2}{4}\right)^x + 4 \right)}$

$$\lim_{x\to+\infty} A(x) = \lim_{x\to+\infty} \left(\left(\frac{3}{4}\right)^x \cdot \frac{\frac{4}{3}}{\left(\frac{2}{4}\right)^x + 4} \right) = 0 \cdot \frac{4}{12} = 0$$

(3) Αν $0 < a < 3$. $A(x) = \dfrac{3^x}{4^x} \cdot \dfrac{1 + \left(\frac{a}{3}\right)^x \cdot \frac{1}{a}}{\left(\frac{2}{4}\right)^x + 1}$

$\lim\limits_{x\to+\infty} \left(\frac{3}{4}\right)^x = 0$. $\lim\limits_{x\to+\infty} \left(\frac{a}{3}\right)^x = 0$

$$\lim_{x\to+\infty} A(x) = 0 \cdot \frac{1+0}{0+1} = 0$$

Σ - Λ (α) $\lim_{x\to 1}\left(x^4+x^3-3\right) = -1 < 0$ Άρα $x^4+x^3-3 < 0$ κοντά στο 1 Σ

Άρα δεν είναι καλώς ορισμένο το όριο.

(β) Για να υπάρχει το όριο πρέπει και αρκεί $\eta\mu\lambda = \lambda \Leftrightarrow \lambda = 0$ Σ

(γ) Λ $f(x) = \frac{2|x|}{x}$

(δ) $\lim_{x\to x_0} f^3(x) = 1 > 0 \Rightarrow f^3(x) > 0$ κοντά στο x_0. $f(x) = \sqrt[3]{f^3(x)}$ Σ

$\lim_{x\to x_0} \sqrt[3]{f^3(x)} = \sqrt[3]{\lim_{x\to x_0} f^3(x)} = \sqrt[3]{1} = 1$.

(ε) Σ π.χ. $f(x) = \frac{|x|}{x}$

(στ) $\lim_{x\to x_0}\left(\underbrace{3f(x)-5}_{A(x)}\right) = 1$ Σ

(ζ) $\lim_{x\to x_0}\left(\sqrt{f^2(x)}\right) = \lim_{x\to x_0}|f(x)| = \left|\lim_{x\to x_0} f(x)\right| = |\lambda| = -\lambda$ Σ

(η) Λ

Scanned with CamScanner