Βασικά τριγωνομετρικά όρια

1) $\lim\limits_{x \to 0} \frac{ημx}{x} = 1$, $\lim\limits_{x \to 0} \frac{συνx - 1}{x} = 0$, $\lim\limits_{x \to 0} \frac{συνx - 1}{x^2} = -\frac{1}{2}$

Απόδειξη (3) $\lim\limits_{x \to 0} \frac{συνx - 1}{x^2} = \lim\limits_{x \to 0} \frac{συν^2x - 1}{x^2 \cdot (συνx + 1)} = \lim\limits_{x \to 0} \left(\frac{-ημ^2x}{x^2} \cdot \frac{1}{συνx + 1} \right) = -\frac{1}{2}$

(2) $\lim\limits_{x \to 0} \frac{συνx - 1}{x} = \lim\limits_{x \to 0} \left(\frac{συνx - 1}{x^2} \cdot x \right) = -\frac{1}{2} \cdot 0 = 0$

$\lim\limits_{x \to 0} \frac{(συνx - 1)^2}{x^3} = 0$ $\quad \lim\limits_{x \to 0} \frac{(συνx - 1)^2}{x^4} = \lim\limits_{x \to 0} \left(\frac{συνx - 1}{x^2} \right)^2 = \frac{1}{4}$

$\lim\limits_{x \to 0^+} \frac{συνx - 1}{x^3} = \lim\limits_{x \to 0^+} \left(\frac{συνx - 1}{x^2} \cdot \frac{1}{x} \right) = -\infty$

$\lim\limits_{x \to 0} \frac{(συνx - 1)^2}{x^4} = \lim\limits_{x \to 0} \left(\frac{(συνx - 1)^2}{x^2} \cdot \frac{1}{x^2} \right) = 0 \cdot \infty$

ασκ. 10 $f(x), g(x) : \mathbb{R} \to (0, +\infty)$, $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} g(x) = 0$

Ν.δ.ο. $\underbrace{\lim\limits_{x \to x_0} \dfrac{f^3(x) + g^3(x)}{f^2(x) + g^2(x)}}_{A(x)} = 0$

$$A(x) = \frac{f^3(x)}{f^2(x) + g^2(x)} + \frac{g^3(x)}{f^2(x) + g^2(x)}.$$

Τώρα
$$0 < \frac{f^3(x)}{f^2(x) + g^2(x)} < \frac{f^3(x)}{f^2(x)} = f(x) \to 0 < \frac{f^3(x)}{f^2(x) + g^2(x)} < f(x)$$

όμως $\lim\limits_{x \to x_0} f(x) = 0$

Απο ΚΠ $\lim\limits_{x \to x_0} \dfrac{f^3(x)}{f^2(x) + g^2(x)} = 0$

Ομοίως $\lim\limits_{x \to x_0} \dfrac{g^3(x)}{f^2(x) + g^2(x)} = 0$ Αρα $\lim\limits_{x \to x_0} A(x) = 0$

Θεώρημα: Όριο σύνθεσης συνάρτησης.

Αν $\lim_{x\to x_0} g(x) = u_0$ και $\lim_{u\to u_0} f(u) = \ell$ τότε $\lim_{x\to x_0} f(g(x)) = \ell$

Να υπολογίσετε τα όρια

$a\neq 0$

$$\lim_{x\to 0}\frac{ημ ax}{x} = \lim_{x\to 0}\left(a\cdot\frac{ημ ax}{ax}\right) = a\cdot\lim_{x\to 0}\frac{ημ ax}{ax} = a\cdot\lim_{u\to 0}\frac{ημ u}{u} = a\cdot 1 = a$$

θέτω $ax = u$

όταν $x\to 0$ τότε $u\to 0$

$$\lim_{x\to 0}\frac{εφx}{x} = \lim_{x\to 0}\left(\frac{\frac{ημx}{συνx}}{\frac{x}{1}}\right) = \lim_{x\to 0}\frac{ημx}{x\cdot συνx} = \lim_{x\to 0}\left(\frac{ημx}{x}\cdot\frac{1}{συνx}\right) = 1\cdot 1 = 1$$

$$\lim_{x\to 0}\frac{εφ^2x}{x} = 0 \qquad \lim_{x\to 0}\frac{εφx}{x^2} = \pm\infty$$

Ασκ. 1 / Σελ. 2

στ) $$\lim_{x\to 0}\frac{x^2 - ημx + 5x}{2x^2 + 3ημx - 2x} = \lim_{x\to 0}\frac{x - \frac{ημx}{x} + 5}{2x + \frac{3ημx}{x} - 2} = \frac{0-1+5}{0+3-2} = \frac{4}{1} = 4$$

$$\lim_{x\to +\infty}\left(x\cdot ημ\frac{1}{x}\right) = \lim_{x\to +\infty}\frac{ημ\frac{1}{x}}{\frac{1}{x}} = \dots$$

$0\cdot ημ\infty$

$$\lim_{x\to 0}\left(x\cdot ημ\frac{1}{x}\right)$$

$$\left|x\,ημ\frac{1}{x}\right| = |x|\cdot\left|ημ\frac{1}{x}\right| \leq |x|\cdot 1$$

$$\left|x\,ημ\frac{1}{x}\right| \leq |x| \Leftrightarrow -|x| \leq x\,ημ\frac{1}{x} \leq |x|$$

1) Αν $\lim\limits_{x \to x_0} f(x) = 0$, τότε $\lim\limits_{x \to x_0} (\eta\mu f(x)) = 0$ Σ

$$|\eta\mu f(x)| \leq |f(x)| \Rightarrow -|f(x)| \leq \eta\mu f(x) \leq |f(x)|$$

κπ.

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