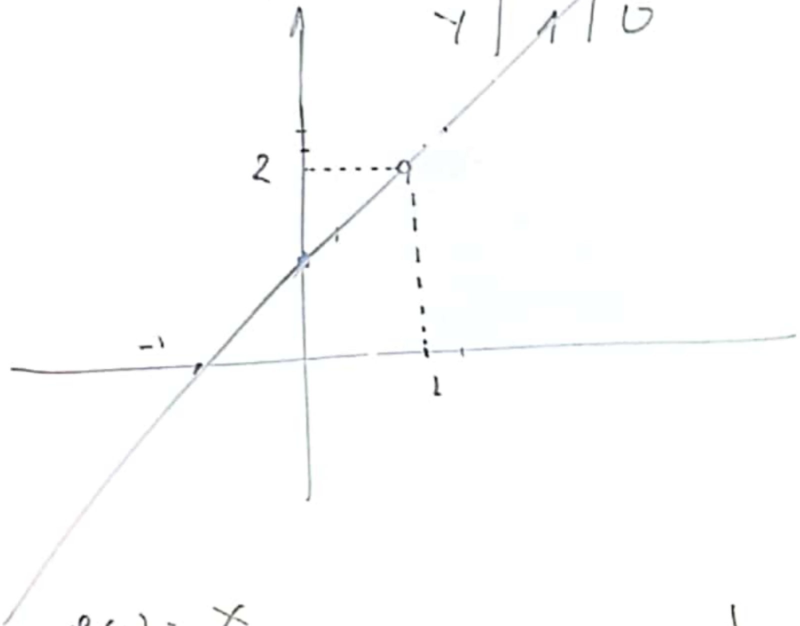
Σελ 40

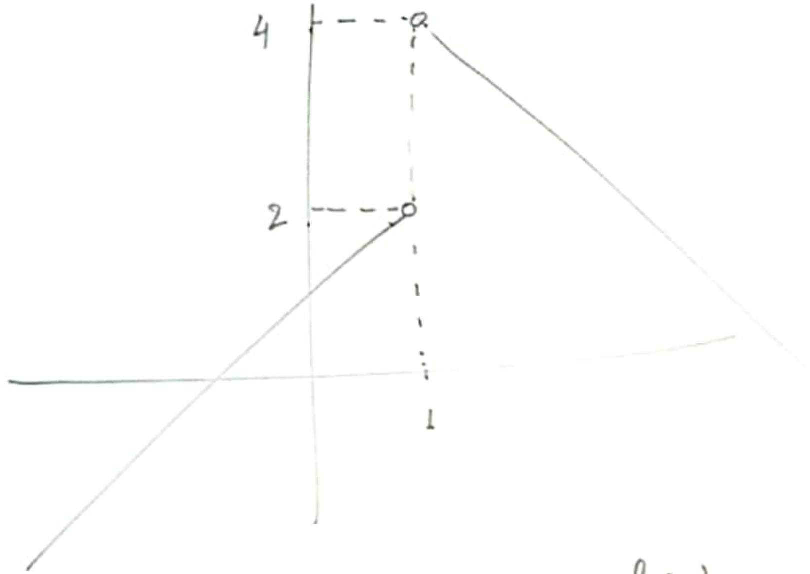
$$f(x) = \frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x+1$$

$$D_f = \mathbb{R} - \{1\}$$

x	0	-1
y	1	0



$$f(x) = \begin{cases} x+1, & x<1 \\ -x+5, & x>1 \end{cases}$$



$f(1) = \times$

$f(1{,}1) = 2{,}1$
$f(1{,}01) = 2{,}01$
$f(1{,}001) = 2{,}001$

$$\lim_{x \to 1^+} f(x) = 2$$

$f(0{,}9) = 1{,}9$
$f(0{,}99) = 1{,}99$
$f(0{,}999) = 1{,}999$

$$\lim_{x \to 1^-} f(x) = 2$$

$\nexists\, f(1)$

$$\exists \lim_{x \to 1} f(x) = 2$$

$$\lim_{x \to 1^+} f(x) = 4$$

$$\lim_{x \to 1^-} f(x) = 2$$

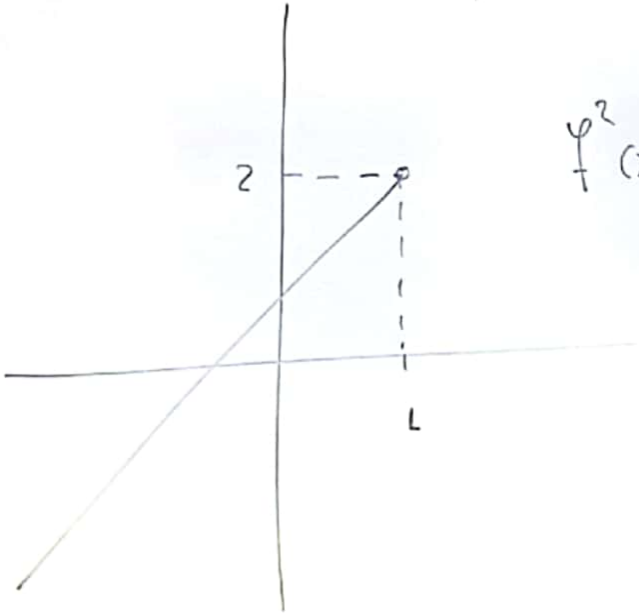
$\nexists\, f(1)$

$$\nexists \lim_{x \to 1} f(x)$$

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ 1, & x \notin \mathbb{Q} \end{cases}$$

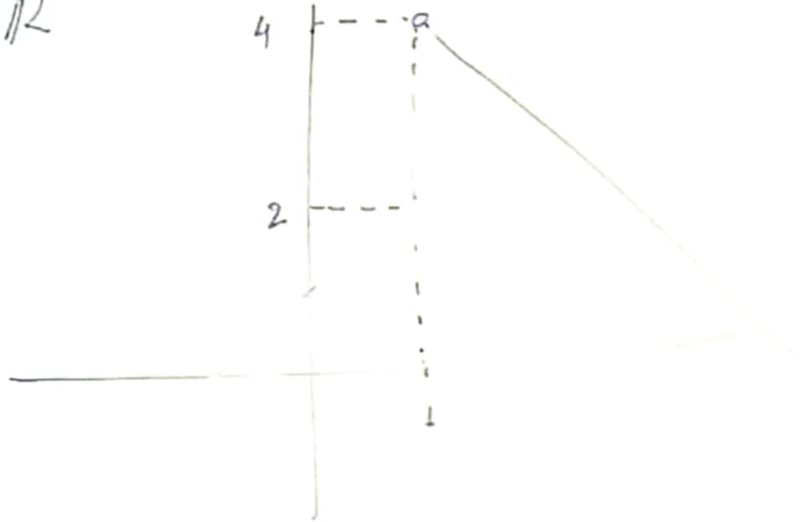
$$f(x) = \frac{|x|}{x} \begin{cases} x>0 & \frac{x}{x} = 1 \\ x<0 & \frac{-x}{x} = -1 \end{cases}$$

$$\left.\begin{aligned} \lim_{x\to 0^+} f(x) &= 1 \\ \lim_{x\to 0^-} f(x) &= -1 \end{aligned}\right\} \nexists \lim_{x\to 0} f(x)$$



$$f^2(x) = \frac{|x|^2}{x^2} = \frac{x^2}{x^2} = 1, \quad x \in \mathbb{R}^*$$

$$\lim_{x\to 0} f^2(x) = 1$$



$D_f = (-\infty, 1)$

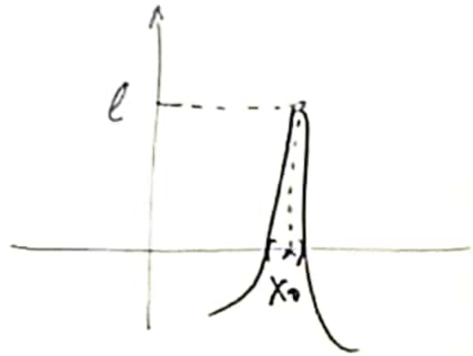
$\lim_{x\to 1} f(x) = \lim_{x\to 1^-} f(x) = 2$

$D_f = (1, +\infty)$

$$\left.\begin{aligned} \lim_{x\to 1^+} f(x) &= 4 \\ \lim_{x\to 1^-} f(x) &= ox \end{aligned}\right\} \lim_{x\to 1} f(x) = 4$$

Scanned with CamScanner

Ιδιότητες των ορίων.



1) Αν $\lim\limits_{x\to x_0} f(x) = \ell > 0$ τότε $f(x) > 0$ κοντά στο x_0

2) Αν $\lim\limits_{x\to x_0} f(x) = \ell < 0$ 〃 $f(x) < 0$ 〃 〃

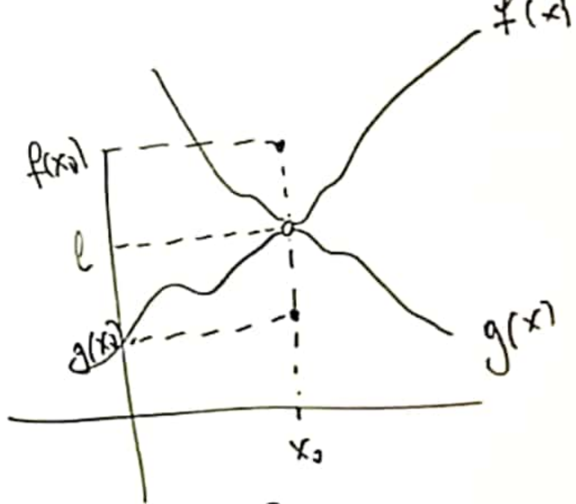
3) Αν $\lim\limits_{x\to x_0^+} f(x) = \ell > 0$ 〃 $f(x) > 0$ 〃 στο x_0^+

Θεώρημα: Αν f, g έχουν όριο στο x_0 και $f(x) \leq g(x)$ κοντά στο x_0, τότε

$$\lim_{x\to x_0} f(x) \leq \lim_{x\to x_0} g(x)$$

$$f(x) \geq 0 \to \lim_{x\to x_0} f(x) \geq 0$$

Σχόλιο: Αν $f(x) < g(x)$ τότε $\lim\limits_{x\to x_0} f(x) \leq \lim\limits_{x\to x_0} g(x)$



$f(x) > g(x) \quad \forall x \in \mathbb{R}$

$\lim\limits_{x\to x_0} f(x) = \lim\limits_{x\to x_0} g(x) = \ell$

Σελ.48

1) $\lim_{x\to x_0}\left(f(x)\overset{\div}{\overset{\cdot}{\pm}}g(x)\right)=\lim_{x\to x_0}f(x)\overset{\div}{\overset{\cdot}{\pm}}\lim_{x\to x_0}g(x)$

2) $\lim_{x\to x_0}\left(c\cdot f(x)\right)=c\cdot\lim_{x\to x_0}f(x)$

3) $\lim_{x\to x_0}\left(f(x)\right)^{\nu}=\left(\lim_{x\to x_0}f(x)\right)^{\nu}$

4) $\lim\sqrt[\nu]{f(x)}=\sqrt[\nu]{\lim f(x)}$

5) $\lim\left|f(x)\right|=\left|\lim f(x)\right|$

Μπορεί να $\nexists \lim_{x\to x_0}f(x)$, $\lim_{x\to x_0}g(x)$

και να $\exists \lim_{x\to x_0}\left(f(x)+g(x)\right)$

π.χ. $f(x)=\frac{|x|}{x}$, $g(x)=1-\frac{|x|}{x}$

$\lim_{x\to 0}f(x)\longrightarrow\nexists$

$f(x)+g(x)=1,\quad \forall x\in\mathbb{R}^*$

$\lim_{x\to 0}\left(f(x)+g(x)\right)=1$

1) Αν $\lim_{x\to x_0}f^2(x)=\ell>0$ τότε $\lim_{x\to x_0}f(x)=\sqrt{\ell}$ ή $\lim_{x\to x_0}f(x)=-\sqrt{\ell}$ Λ

2) Αν $\lim_{x\to x_0}f^2(x)=\ell>0$ και υπάρχει το $\lim_{x\to x_0}f(x)$, τότε $\lim_{x\to x_0}f(x)=\sqrt{\ell}$ ή $-\sqrt{\ell}$ Σ