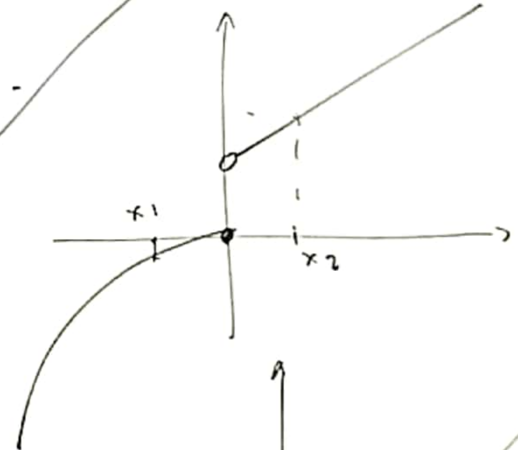
ασκ.8 | 2° φυλ. $f(x)=\dfrac{x}{1+|x|}$, $g(x)=\dfrac{x}{1-|x|}$

$A_{g\circ f}=\mathbb{R}$ (χθες)

$$(g\circ f)(x)=g(f(x))=g\left(\frac{x}{1+|x|}\right)=\frac{\frac{x}{1+|x|}}{1-\left|\frac{x}{1+|x|}\right|}=\frac{\frac{x}{1+|x|}}{\frac{1+|x|}{1+|x|}-\frac{|x|}{1+|x|}}=$$

$$\frac{\frac{x}{1+|x|}}{\frac{1+\cancel{|x|}-\cancel{|x|}}{1+|x|}}=\frac{x\cdot\cancel{(1+|x|)}}{\cancel{1+|x|}}=x.$$



$$f(x)=\begin{cases}-x^2, & x\le 0\\ x+1, & x>0\end{cases}$$

ν.δ.ο. $f\uparrow$ στο $\mathbb{R}$

$\boxed{1^\circ}$ $x_1<x_2\le 0$ .... $f(x_1)<f(x_2)$

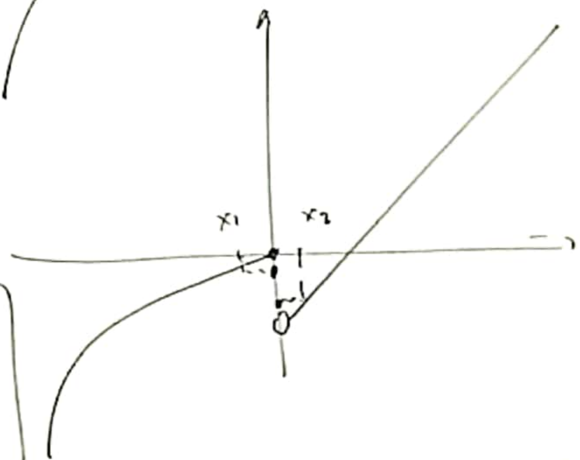
$\boxed{2^\circ}$ $0<x_1<x_2$ .... $f(x_1)<f(x_2)$

$\boxed{3^\circ}$ $x_1\le 0<x_2$

$x_1\le 0 \overset{f\uparrow}{\underset{(-\infty,0]}{\Rightarrow}} f(x_1)\le f(0)=0$

$x_2>0 \Rightarrow x_2+1>1 \Rightarrow f(x_2)>1$

$\Rightarrow f(x_1)<f(x_2)$ $\Big\}$ $f\uparrow$ $\mathbb{R}$



$x_1<x_2$

$f(x_1)>f(x_2)$

ασκ.9 | 2° φυλ. Αν $f^3(x)=x$, $\forall x\in\mathbb{R}$ να ορίσετε την $f\circ f$.

Αν $x\geq 0$: $f^3(x)=x \Rightarrow f(x)=\sqrt[3]{x}$

Αν $x<0$: $f^3(x)=x \Rightarrow f(x)=-\sqrt[3]{-x}$

$$\Rightarrow f(x)=\begin{cases}\sqrt[3]{x}, & x\geq 0\\ -\sqrt[3]{-x}, & x<0\end{cases} \quad D_f=\mathbb{R}.$$

$$D_{f\circ f}=\{x\in D_f \,/\, f(x)\in D_f\}=\mathbb{R}.$$

$$(f\circ f)(x)=f(f(x)) \xrightarrow{\text{αν } x\geq 0} \sqrt[3]{f(x)}=\sqrt[3]{\sqrt[3]{x}}=\sqrt[9]{x}.$$

$$(f\circ f)(x)=f(f(x)) \xrightarrow[x<0]{\text{αν}} -\sqrt[3]{-f(x)}=-\sqrt[3]{-\left(-\sqrt[3]{-x}\right)}=$$

$$=-\sqrt[3]{\sqrt[3]{-x}}=-\sqrt[9]{-x}.$$

εξετάζω πότε $f(x)\geq 0$

α) Για $x\geq 0$ έχω $\sqrt[3]{x}\geq 0$ ισχύει $f(x)\geq 0$

β) Για $x<0 \Rightarrow$ $-x>0 \Rightarrow$ $\sqrt[3]{-x}>0 \Rightarrow$ $-\sqrt[3]{-x}<0$ $f(x)<0$

Άρα
$$(f\circ f)(x)=\begin{cases}\sqrt[9]{x}, & x\geq 0\\ -\sqrt[9]{-x}, & x<0\end{cases}$$