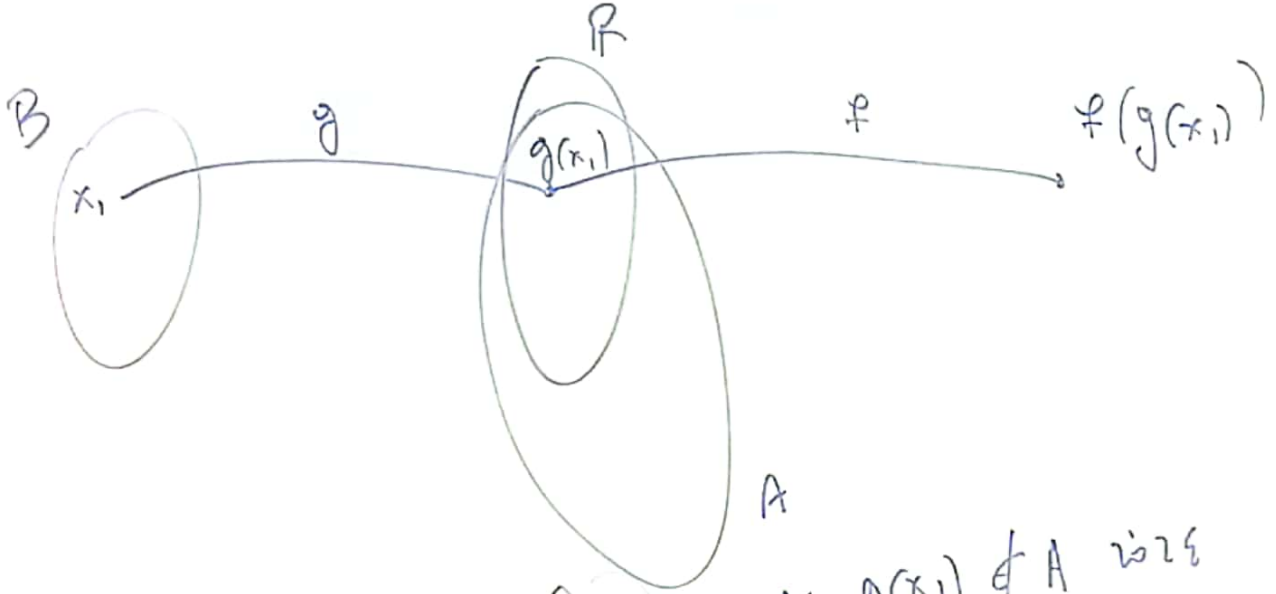
Σύνθεση συναρτήσεων.

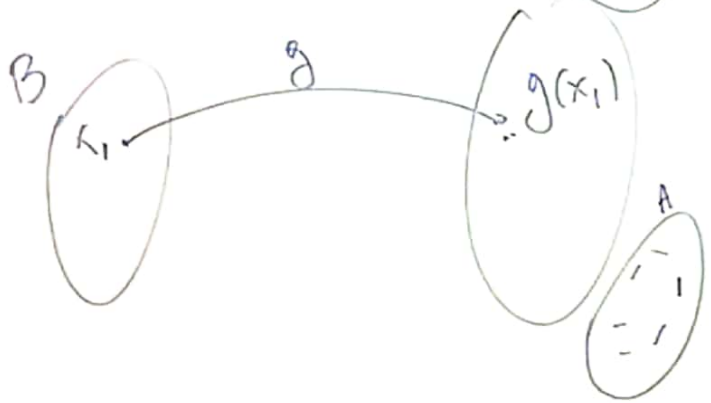
$f: A \longrightarrow \mathbb{R}$, $g: B \longrightarrow \mathbb{R}$

1) Π.Ο. της $f \circ g$. $D_{f\circ g} = \{ x \in B : g(x) \in D_f \}$

2) $(f \circ g)(x) = f(g(x))$ $\forall x \in D_{f\circ g}$.



Αν $g(x_1) \notin A$ τότε δεν ορίζεται η σύνθεση



$f(x) = \sqrt{x}$, $x \geq 0$, $g(x) = x^2$, $x \in \mathbb{R}$

Να ορίσετε την $g \circ f$

$D_f = [0, +\infty)$, $D_g = \mathbb{R}$. $D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} = [0, +\infty)$

$x \geq 0$ $\quad \sqrt{x} \in \mathbb{R}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x$.

1) Δεν ισχύει η αντιμεταθετική ιδιότητα $g \circ f \neq f \circ g$

2) Ισχύει η προσεταιριστική $\rightarrow$ $f \circ (g \circ h) = (f \circ g) \circ h$

ασκ.1 $g(x)=\sqrt{1-x^2}$. Να ορίσετε την $g\circ g$.

Η g ορίζεται όταν

$1-x^2 \geq 0 \iff$

$-x^2 \geq -1 \iff$

$x^2 \leq 1 \iff$

$|x| \leq 1 \iff$

$-1 \leq x \leq 1$

Η $g\circ g$ ορίζεται όταν

$x \in D_g$ και $g(x) \in D_g$

$-1 \leq x \leq 1$ $\quad -1 \leq \sqrt{1-x^2} \leq 1 \iff$

$\sqrt{1-x^2} \leq 1 \iff 1-x^2 \leq 1 \iff -x^2 \leq 0$ ΙΣΧΥΕΙ

$D_{g\circ g} = [-1, 1]$

$$g(g(x)) = \sqrt{1-\sqrt{1-x^2}^2} = \sqrt{1-1+x^2} = \sqrt{x^2} = |x|$$

ασκ. 2 $\Delta = [-1, 2]$

$g(x) = f(2x-1)$

πρέπει $x \in \mathbb{R}$  $2x-1 \in [-1, 2]$

$-1 \leq 2x-1 \leq 2 \Leftrightarrow$

$0 \leq 2x \leq 3 \Leftrightarrow$

$0 \leq x \leq 3/2$

$h(x) = 2x-1, \quad x \in \mathbb{R}$

$g(x) = (f \circ h)(x)$

$x \in D_h$ και $h(x) \in D_f$

$F(x) = g(g(x))$

$x \in D_g$  $g(x) \in D_g$

$-1 \leq 2x-1 \leq 2$

$-1 \leq x \leq 2$

Ασκ. 3

$(g \circ f)(x) = x^2 - x, \quad x \in \mathbb{R}$

α) $g(x) = 3 - 5x, \quad f = ?$

$g(f(x)) = x^2 - x \iff$

$3 - 5f(x) = x^2 - x \iff$

$-5f(x) = x^2 - x - 3 \iff$

$f(x) = \dfrac{-x^2 + x + 3}{5} \quad x \in \mathbb{R}$

(b) $f(x) = 2x - 5, \quad g = ?$

$g(f(x)) = x^2 - x \iff$

$g(2x - 5) = x^2 - x$

Θέτω $u = 2x - 5 \iff u + 5 = 2x \iff x = \dfrac{u+5}{2}$

$g(u) = \left(\dfrac{u+5}{2}\right)^2 - \dfrac{u+5}{2}$

$g(x) = \left(\dfrac{x+5}{2}\right)^2 - \dfrac{x+5}{2}$

ασκ. 6/

(iii) $(g \circ f)(x) = |\sin x| \quad \forall x \in \mathbb{R} \qquad g(x) = \sqrt{1-x^2}, \qquad f = ?$

$g(f(x)) = |\sin x| \Leftrightarrow$

$\sqrt{1-f^2(x)} = |\sin x| \Leftrightarrow$

$1 - f^2(x) = \sin^2 x \Leftrightarrow$

$-f^2(x) = -1 + \sin^2 x \Leftrightarrow$

$f^2(x) = 1 - \sin^2 x \Leftrightarrow$

$f^2(x) = \cos^2 x \Leftrightarrow$

$|f(x)| = |\cos x| \Leftrightarrow \quad f(x) = \cos x \quad \text{ή} \quad f(x) = -\cos x$

$$f(x) = \begin{cases} \cos x, & x \in A \\ -\cos x, & x \in \mathbb{R} - A \end{cases}$$