(1) (i) $f(x) = \dfrac{2x+3}{x^2-4x+\lambda}$

$x^2 - 4x + \lambda \neq 0 \quad \forall x \in \mathbb{R}$

$\Leftrightarrow \Delta < 0 \Leftrightarrow$

$(-4)^2 - 4 \cdot 1 \cdot \lambda < 0 \Leftrightarrow$

$16 - 4\lambda < 0 \Leftrightarrow$

$4\lambda > 16$

$\lambda > 4$

$x^2 - 4x \neq -\lambda$

$x^2 - 4x + 4 \neq 4 - \lambda$

$(x-2)^2 \neq 4 - \lambda$

ii) $f(x) = \dfrac{x}{m\mu x - \lambda}$

$m\mu x - \lambda \neq 0 \Leftrightarrow \forall x \in \mathbb{R}$

$\Rightarrow$

$m\mu x \neq \lambda$

$\Leftrightarrow \lambda \in (-\infty, -1) \cup (1, +\infty)$

Ασκ. 2 $y_1 \in f(A) \Leftrightarrow \exists x_1 \in D_f : f(x_1) = y_1$

i) $2 \stackrel{?}{\in} f(A) \quad f(x) = \dfrac{3x-1}{x^2+1}$

Εξετάζω αν $\exists x_1 \in D_f : f(x_1) = 2$

Άρα πρέπει εξίσωση $f(x) = 2 \Leftrightarrow$

$\dfrac{3x-1}{x^2+1} = 2 \Leftrightarrow 3x - 1 = 2x^2 + 2 \Leftrightarrow 2x^2 - 3x + 3 = 0$

$\Delta = 9 - 4 \cdot 2 \cdot 3 = 9 - 24 < 0$

Άρα $2 \notin f(A)$

Ασκ.2 / iii) $f(x) = \eta\mu x + \sigma\upsilon\nu x$

$1,9 \overset{?}{\in} f(\mathbb{R})$

$\perp \qquad \perp$

$\eta\mu x \qquad \sigma\upsilon\nu x$

$(1^2+1^2)\cdot(\eta\mu^2 x + \sigma\upsilon\nu^2 x) \geq (\eta\mu x + \sigma\upsilon\nu x)^2$

$2 \geq (\eta\mu x + \sigma\upsilon\nu x)^2$

$-\sqrt{2} \leq \eta\mu x + \sigma\upsilon\nu x \leq \sqrt{2}$

Να δ.ο.

$(x^2+y^2)\cdot(a^2+b^2) \geq (ax+by)^2$

$(ax-by)^2 \geq 0$

$x \quad y$

$a \quad b$